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Synchronization of coupled space-clamped fitzhugh-nagumo neurons via an adaptive integral type of terminal sliding mode control

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Abstract. In this study, an integral type of terminal sliding mode is introduced to develop the robust adaptive control scheme for achieving the state synchronization between two coupled space-clamped FitzHugh-Nagumo neurons with gap junctions and external electrical stimulation by taking account of the external disturbances. Sufficient conditions to guarantee the stable synchronization are given and the proof of theorem is made in the sense of the Lyapunov stability. In addition, numerical simulations are also performed to verify the effectiveness of presented scheme.

Key words. FN neuron, state synchronization, adaptive control.

1. Introduction

External electrical stimulation (EES) is a therapy for cognitive disorders such as Parkinson's disease, epilepsy and dystonia [1]. Investigation of neuronal synchronization has become one of the widely researched problems in the field of neuro science [2-4]. It has attracted many brain researchers over the past decade in order to understand the underlying mechanism of external stimulation and hence to improve the stimulation therapy based treatments for cognitive diseases.

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Neuron is the fundamental element of every nervous system. Dynamic behavior of neuron is widely studied to find the mechanism of neuronal spiking for effective neurotransmission and brain signal proceeing. In the field of neuroscience, chaotic phenomenon and other complex behavior of neurons such as bifurcation, periodic, and quasi-periodic were studied based on the various nerve modes, such as, Hodglein-Huxley (HH) neuron model [5], FitzHugh-Nagumo (FN) neuron model [6, 7], and Hindmarsh-Rose model [8]. Due to the simplicity, space-clamped FN neuron model [9], which being the cable model of cylindrical cell, is one simplified model of the HH neuron model to describe the neuron dynamics and utilized to study neural firings.

Without control, identical coupled neurons can eventually synchronize only when the coupling strength is above a certain critical value [10, 11], which may be beyond the physiological condition. In biological experiments, the synchronization of two coupled neurons can be achieved when depolarized by an external DC current. Motivated by [10], synchronization of chaotic neurons under EES is attracted many interests during the past decades.

For two coupled space-clamped FN neurons with EES, various control schemes have been successfully applied to control and synchronization. In [9], the LMI-based robust adaptive synchronized scheme was introduced. In [11], the dynamic synchronization of two coupled space-clamped FN neurons with gap junctions was addressed and the robust performance was ensured against bounded disturbances. In [10], a robust adaptive sliding-mode controller instead of the active elimination of nonlinear dynamics was proposed to achieve the chaotic synchronized problem in the presence of system uncertainty and external disturbances between two uncoupled FN neurons with different ionic currents and ESS.

Motivated by [10], by taking account the external disturbances, a novel adaptive integral type of terminal sliding mode control for synchronization of two coupled space-clamped FN neurons with sinusoidal ESS is developed in this study. The developed controller, which can achieve the state synchronization of two coupled FN neurons, associated with time varying feedback gains can not only tackle the external disturbances but also compensate for the mismatch nonlinear dynamics of synchronized error system without direct cancellation of nonlinear terms. Meanwhile, according to the novel integral type of adaptive terminal sliding mode, these feedback gains are not determined in advance but updated by the adaptive rules. Sufficient conditions to guarantee the stable synchronization are given in the sense of the Lyapunov stability. In addition, numerical simulations are also performed to verify the effectiveness of presented scheme.

The rest of this study is organized as follows. The formulation of problem for state synchronization between two coupled FN neurons and the design of adaptive controller are demonstrated in Section 2 and 3, respectively. In Section 4, numerical simulations are performed to show the effectiveness of the proposed controller. In the final section, some concluding remarks are made.

2. Formulation of the problem for state synchronization

The general form of two coupled space-clamped FN neuron subject to EES under external disturbances is described by the following second-order non-autonomous differential equations :

$$\text{Master FN neuron}: \begin{cases} \dot{x}_1 = x_1(x_1-1)(1-\alpha x_1) - x_2 + \delta(x_1 - y_1) + f\cos(\omega t) \\ \qquad +d_m(t) \\ \dot{x}_2 = \beta x_1 - \gamma x_2 \end{cases}$$

$$\text{Slave FN neuron}: \begin{cases} \dot{y}_1 = y_1(y_1-1)(1-\alpha y_1) - y_2 + \delta(y_1 - x_1) + f\cos(\omega t) \\ \qquad +d_s(t) + \phi(t) \\ \dot{y}_2 = \beta y_1 - \gamma y_2 \end{cases}$$

(1)

where $x_1,\ x_2$ and $y_1,\ y_2$ are the normalized state variables of the master and slave systems, respectively. The parameter δ represents the strength gap junction between the master and slave FN neurons, and $f\cos(\omega t)$ represents the EES current with angular frequency ω at time t . $d_m(t),\ d_s(t)$ are the external disturbance, and $\phi(t)$ is the control to be determined. In general, the external disturbances $d_m(t),\ d_s(t)$ are assumed to be bounded as follows:

$0 \le |d_m(t)| \le D_1, \quad 0 \le |d_s(t)| \le D_2,\ \forall t$ (2)

where D_1 and D_2 are positive constants. System parameters of the coupled model in (1) are set to

$\alpha = 10,\ \beta = 1,\ \gamma = 0.1,\ \delta = 0.01,\ \omega = 0.28\pi,\ f = 5/\,(14\pi)$ (3)

Figures 1 and 2 shows the behaviors of two coupled chaotic space-clamped FN neurons with the initial conditions $(x_1(0),\ x_2(0)) = (-0.1,\ -0.1)$, $(y_1(0),\ y_2(0)) = (0.3,\ 0.3)$.

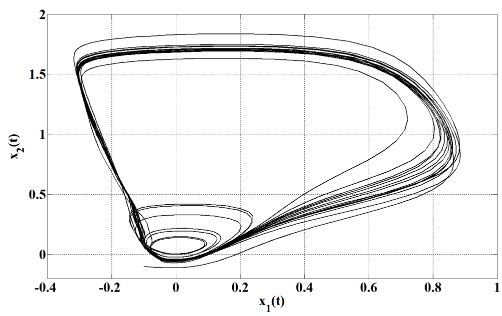

Fig. 1. State trajectory of master neuron

To proceed, the synchronized error states between systems in (1) are defined as

$e_1(t) = x_1(t) - y_1(t), \quad e_2(t) = x_2(t) - y_2(t).$ (4)

Taking the time derivative of (4), the synchronized error system can be expressed in the following

$$\begin{cases} \dot{e}_1 = [-\alpha f_1(x_1,x_2) + f_2(x_1,x_2) - (1+2\delta)]e_1 - e_2 + d_m(t) - d_s(t) + \phi(t) \\ \dot{e}_2 = \beta e_1 - \gamma e_2 \end{cases} \quad (5)$$

where the functions $f_1(x_1,x_2) = x_1^2 + x_1x_2 + x_2^2$ and $f_2(x_1,x_2) = (1+\alpha)(x_1+x_2)$ are nonlinear functions represented the mismatch dynamics and bounded because

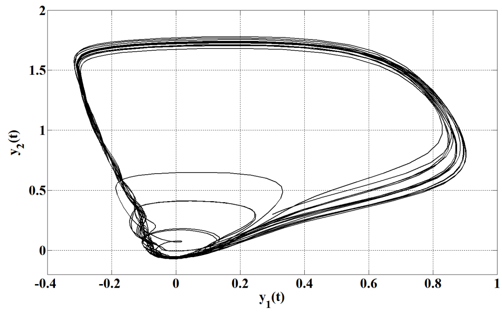

Fig. 2. State trajectory of slave neuron

of the bounded phase trajectories of $x_1(t)$, $x_2(t)$. To this end, it is clear that the problem of state synchronization is replaced by the equivalent of stabilizing the synchronized error system (5) by utilizing an appropriate control input $\phi(t)$. The goal of the current problem is to design the control $\phi(t)$ such that $\lim_{t\to\infty} e_1(t) \to 0$ and $\lim_{t\to\infty} e_2(t) \to 0$ for any initial conditions of the synchronized error system (5). It means that the state behaviors of the slave neuron can tend to ones of the master neuron.

3. Design of Adaptive Integral Type of Terminal Sliding Mode Control

The design approach of robust adaptive sliding mode controller involves two steps. (1) The appropriate sliding surface for desired sliding motion is selected. In the sliding surface, the slave neuron will be synchronous with the master neuron. (2) The robust controller $\phi(t)$ is designed that brings any trajectory in phase space of the error dynamics to and stay in the sliding surface even in the events of external disturbances $d_m(t)$, $d_s(t)$.

The novel integral type of terminal sliding mode surface is defined as follows.
$\sigma(t) = \frac{1}{\lambda}\left[e_2(t) + \gamma \int_{\tau=0}^{t} e_2(\tau)d\tau\right] + [e_1(t)]^{\frac{p}{q}}$ (6)

where $\lambda > 0$, $1 < p/q < 2$, p and q are positive odd integrals. For the existence of the sliding mode, it is necessary and sufficient that $\sigma(t) = 0$ and $\dot{\sigma}(t) = 0$. Therefore, the sliding mode dynamics are determined by the following nonlinear differential equation:

$\dot{\sigma}(t) = 0 \Rightarrow \frac{1}{\lambda}[\dot{e}_2(t) + \gamma e_2(t)] + \frac{p}{q}[e_1(t)]^{\frac{p}{q}-1}\dot{e}_1(t) = 0$ (7)

The finite time T_s , which is taken to travel from $e_1(T_r) \neq 0$ to $e_1(T_r + T_s) = 0$, is given by

$T_s = \frac{\lambda p}{\beta(p-q)}[e_1(0)]^{\frac{p}{q}-1}$ (8)

where T_r is the time of synchronized error state trajectory reaching to the sliding surface $\sigma(t) = 0$. In the following, the control $\phi(t)$ of system (5) for achieving the state synchronization is proposed.

Theorem. If the control law $\phi(t)$ in system (5) is taken as follows:

$\phi(t) = -\frac{\beta q}{\lambda p}\left[e_1(t)\right]^{2-(p/q)} + e_2(t) - \left[K_0(t) + K_1(t)\left|e_1(t)\right| + K_2(t)\left|\sigma(t)\right|^a\right]\cdot \mathrm{sgn}(\sigma(t))$ (9)

where $\sigma(t)$ is the sliding surface defined in (6), $0 < a < 1$ is positive design constant and $\mathrm{sgn}(\bullet)$ denotes the sign function. $K_0(t),\ K_1(t)$ and $K_2(t)$ are the adaptive feedback gains updated, respectively, according to the following adaptation algorithms:

$\dot{K}_0(t) = \rho_0\left|\sigma(t)\right|\left|e_1(t)\right|^{(p/q)-1},\quad K_0(0) = 0,\quad \rho_0 > 0$ (10)

$\dot{K}_1(t) = \rho_1\left|\sigma(t)\right|\left|e_1(t)\right|^{(p/q)},\quad K_1(0) = 0,\quad \rho_1 > 0$ (11)

$\dot{K}_2(t) = \rho_2\left|\sigma(t)\right|^{a+1}\left|e_1(t)\right|^{(p/q)-1},\quad K_2(0) = 0,\quad \rho_2 > 0$ (12)

where $\rho_0,\ \rho_1,\ \rho_2$ are the positive adaptation gains determining the adaptation process. Then, states of the synchronized error system (5) will asymptotically approach to and stay in the sliding surface $\sigma(t) = 0$.

Proof. The Lyapunov function candidate of the problem is chosen as

$V(t) = \frac{1}{2}\sigma^2(t) + \frac{p}{2q}\sum_{i=0}^{2}\frac{1}{\rho_i}(K_i(t) - \bar{K}_i)^2$ (13)

where $\bar{K}_0,\ \bar{K}_1,\ \bar{K}_2$ are positive constants and satisfied

$\bar{K}_0 > D_1 + D_2,\quad \bar{K}_1 > 1 + 2\delta + \alpha\left|f_1\right| + \left|f_2\right| > 0,\quad \bar{K}_2 > 0$ (14)

Taking the time derivative of (13) along with the solutions of the synchronized error system (5), the selection of the sliding mode surface (6), and the controller (9), it yields

$$\dot{V} \;=\; \sigma\dot{\sigma} + \frac{p}{q}\sum_{i=0}^{2}\frac{1}{\rho_i}(K_i(t) - \bar{K}_i)\dot{K}_i \qquad (15)$$

where $G = -[\bar{K}_1 - D_1 - D_2]\left|\sigma\right| - [\bar{K}_1 - (1 + 2\delta + \alpha\left|f_1\right| + \left|f_2\right|)]\ \left|e_1\right|\left|\sigma\right| - \bar{K}_2\left|\sigma\right|^{a+1}$. From (15), since $V(t)$ is a positive definite and decreasing function, it follows that the zero equilibrium point ($\sigma = 0,\ K_0 = \bar{K}_0,\ K_1 = \bar{K}_1,\ K_2 = \bar{K}_2$) would be asymptotically stable. It means the states of the synchronized error system (5) will asymptotically approach to and stay in the sliding surface $\sigma(t) = 0$. Once the sliding surface is reached, the time taken to arrive at the equivalent point $e_1(t) = 0$ in the sliding surface is defined in (8). It follows that both the synchronous error states will ultimately tend to zeros. As the control design meets the requirements depicted in this theorem, the state synchronization between systems (1) is achieved. This completes the proof.

4. Numerical simulations

In the section, the numerical studies are performed to verify effectiveness of the proposed adaptive terminal sliding mode controller. Using the fourth-order Runge-Kutta method with the initial conditions $(x_1(0),\ x_2(0)) = (-0.1,\ -0.1)$, $(y_1(0),\ y_2(0)) = (0.3,\ 0.3)$ and system parameters given in Fig. 1 to ensure the chaotic dynamics of the state variables, the synchronized error system (5) with the controller defined in (9) is numerically solved. The external disturbances are assumed to be $d_m = 0.015\sin(3.5t),\ d_m = 0.06in(3t)$, respectively.

For the robust adaptive sliding mode controller described in (9) associated with

(10) and (12), the positive design constants are chosen as $p = 9$, $q = 7$, $\lambda = 0.1$, $a = 1/2$, $\rho_0 = 0.75$, $\rho_1 = 1.0$, and $\rho_2 = 1.25$ In Figure 3, it is shown that the synchronized error states oscillate irregularly when the controller is switched off, and when the controller is in action at $t = 80$, both of the synchronized error states converge to zero and the synchronization is achieved. Time responses of the sliding mode, the control signal, and the adaptive feedback gains are depicted in Figures 4 and 5, respectively. It can be seen that the control signal is continuous and chattering free. In Figure 6, time responses of states for coupled space-clamped FN neurons are illustrated. It depicates that the state synchronization is accomplished between the master and slave FN neurons by applying the develpoed adaptive control scheme.

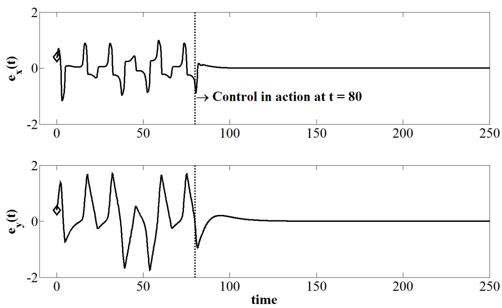

Fig. 3. Time responses of error states

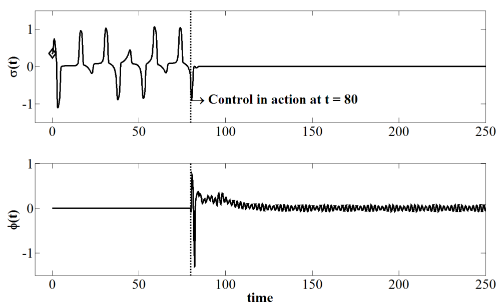

Fig. 4. Time responses of $\sigma(t)$, $\phi(t)$

5. Conclusions

In this study, by defining an integral type of terminal sliding mode, a robust adaptive control law has bee addressed to achieve the state synchronization between two coupled space-clamped FN neurons with gap junctions and external electrical stimulation in the present of external disturbances. Sufficient conditions to guarantee the stability are given based on the Lyapunov stability theorem. Besidies, numerical simulations are also performed to verify the effectiveness of presented scheme. It is

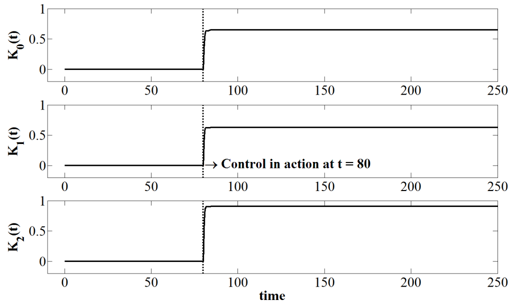

Fig. 5. Time responses of feedback gains

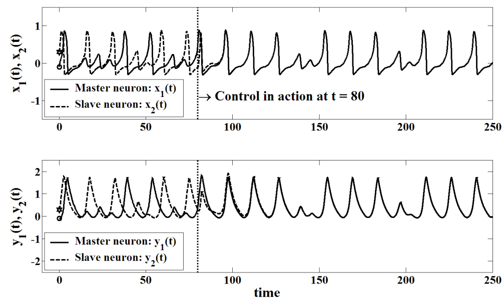

Fig. 6. Time responses of states for master and slave FN neurons

shown that the state synchronization between the master and slave FN neurons is achieved by utlilizing the proposed adaptive integral type of terminal sliding mode control scheme.

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